ABSTRACT

Traditional imaging systems directly image a 2D object plane onto the sensor. Plenoptic imaging systems contain a lenslet array at the conventional image plane and a sensor at the back focal plane of the lenslet array. In this configuration the data captured at the sensor is not a direct image of the object. Each lenslet effectively images the aperture of the main imaging lens at the sensor. Therefore the sensor data retains angular light-field information which can be used for a posteriori digital computation of multi-angle images and axially refocused images. If a filter array, containing spectral filters or neutral density or polarization filters, is placed at the pupil aperture of the main imaging lens, then each lenslet images the filters onto the sensor. This enables the digital separation of multiple filter modalities giving single snapshot, multi-modal images. Due to the diversity of potential applications of plenoptic systems, their investigation is increasing. As the application space moves towards microscopes and other complex systems, and as pixel sizes become smaller, the consideration of diffraction effects in these systems becomes increasingly important. We discuss a plenoptic system and its wave propagation analysis for both coherent and incoherent imaging. We simulate a system response using our analysis and discuss various applications of the system response pertaining to plenoptic system design, implementation and calibration.

Keywords: Plenoptic system, computational imaging, wave propagation, optical design, calibration, optical response.

1. INTRODUCTION

Traditional imaging systems contain a sensor at the image plane. An image of the object is directly captured at the sensor. In a plenoptic system, however, a lenslet array is placed at the conventional image plane and the sensor is at the back focal plane of the lenslet array. Each lenslet therefore images the pupil aperture of the main lens at the sensor. The different sensor pixels sampling different portions of the aperture contain angular information about a given point in the object plane. Therefore, demultiplexing the pixels that contain information from different angles enables multi-angle imaging [1,2]. Appropriate shift and addition of these multi-view images permits refocusing and depth mapping [1,2]. If an array of spectral filters, neutral density filters, polarization filters, etc. is placed at the pupil of the main lens, this system also permits single snap-shot multispectral or multi-modal imaging [3].

Optical design and characterization of plenoptic systems has been discussed infrequently in literature [4,5], and system requirements may depend greatly on the application. For instance, a system used for depth imaging may need to have high angular resolution, but careful focusing may be more critical for multi-spectral imaging. Therefore, we need analysis tools to understand how factors like the axial positions of various elements of a plenoptic system, lens parameters, wavelengths, aberrations, etc. affect system performance.

In this paper we performed a wave propagation analysis for a general plenoptic system and derived the system response for coherent and incoherent imaging. The coherent case is most useful for lens design and characterization of the system. We also discuss applications of the coherent analysis, which was briefly introduced in [4].

2. WAVE ANALYSIS

We consider a system where the object is imaged by a main lens. This main lens may be a camera lens, microscope objective or other imaging system. In our analysis the main lens is considered, for simplicity, as a single thin lens. The spatial coordinates for the object plane are given by \((\xi, \eta)\), and the main lens pupil, by \((x,y)\). The image of the object produced by this main lens is called the primary image. The lenslet array is placed at the plane of this primary image. The coordinates of the lenslet array plane are denoted by \((u,v)\). The main lens is at a distance \(z_1\) from the object, and the lenslet array is placed at a distance \(z_2\) from the plane of the main lens. The sensor is placed at a distance \(z_3\) from the
lenslet array and its coordinates are given by \((t,w)\). The focal lengths of the main lens and the lenslet array are \(f_1\) and \(f_2\) respectively. Their diameters are \(D_1\) and \(D_2\), respectively.

![Diagram of imaging sub-systems](image.png)

**Figure 1. Plenoptic system contains two sub-systems [4,6]**

The plenoptic system, shown in Figure 1, comprises of two sub-systems. The first imaging sub-system consists of the object and the main lens which produces the primary image of the object. The lenslet array is placed at this primary image plane. Each lenslet then images the pupil of the main lens at the sensor plane. The pupil aperture, the lenslet and the sensor comprise the second imaging sub-system. The first and second imaging sub-systems partially overlap with the space \(z_2\) in common.

In this manuscript we refer to the replica of the object produced by the main lens as “primary image”, the data captured by the sensor behind a single lenslet as “pupil image” and the overall sensor data as “plenoptic image” in order to avoid confusion with the general term “image”.

### 2.1 Coherent imaging

In order to analyze this optical layout and its impact on the data collected at the sensor we perform a wave analysis of this system. We denote the generalized pupil function for the main lens by \(P_1\). Its impulse response can be written as,

\[
h_1(u,v;\xi,\eta) = \frac{e^{j\beta_1}}{\lambda^2 z_1 z_2} \exp\left[\frac{jk}{2z_2} (u^2 + v^2)\right] \exp\left[\frac{jk}{2z_1} (\xi^2 + \eta^2)\right] \int dxdy P_1(x,y) \exp\left[\frac{jk}{2} \frac{1}{z_1 + 1/z_2 - 1/f_1} (x^2 + y^2)\right] \exp\left\{-\frac{jk}{z_1} \left[ (u - \mathcal{M} \xi) x + (v - \mathcal{M} \eta) y \right]\right\},
\]

where \(\lambda\) is the wavelength of imaging, \(k = 2\pi/\lambda\) and the magnification from the object to primary image plane is given by \(\mathcal{M} = -z_2/z_1\). Substituting \(x' = x/\lambda z_2\) and \(y' = y/\lambda z_2\) in Eq. (1) and defining the term \(h'_1\) for convenience as the Fourier transform of \(R(x\lambda z_2, y\lambda z_2)\) we get,

\[
h_1(u,v;\xi,\eta) = e^{j\beta_1} \exp\left[\frac{jk}{2z_2} (u^2 + v^2)\right] \exp\left[\frac{jk}{2z_1} (\xi^2 + \eta^2)\right] h'_1(u - \mathcal{M} \xi, v - \mathcal{M} \eta).
\]

Using Eq. (2) and substituting \(\xi' = \mathcal{M} \xi\) and \(\eta' = \mathcal{M} \eta\), we obtain the primary image field for an object having a complex field, \(U_o\), imaged at the plane of the lenslet array by the first imaging sub-system as,

\[
U_i(u,v) = \frac{e^{j\beta_1}}{\mathcal{M}^2} \exp\left[\frac{jk}{2z_2} (u^2 + v^2)\right] \int d\xi' d\eta' U_o\left(\frac{\xi'}{\mathcal{M}}, \frac{\eta'}{\mathcal{M}}\right) \exp\left[\frac{jk}{2z_1} \left[\left(\frac{\xi'}{\mathcal{M}}\right)^2 + \left(\frac{\eta'}{\mathcal{M}}\right)^2\right]\right] h'_1(u - \xi', v - \eta')
\]

\[
= \frac{e^{j\beta_1}}{\mathcal{M}^2} \exp\left[\frac{jk}{2z_2} (u^2 + v^2)\right] \left[ U_o\left(\frac{u}{\mathcal{M}}, \frac{v}{\mathcal{M}}\right) \exp\left[\frac{jk}{2z_1} \left[\left(\frac{u}{\mathcal{M}}\right)^2 + \left(\frac{v}{\mathcal{M}}\right)^2\right]\right]\right] * h'_1(u,v).
\]
where the symbol ∗ refers to a convolution. Assuming each lenslet has a generalized pupil function, \( P \), and \( M \times N \) such lenslets in the array, the amplitude distribution of the field after the lenslet array is

\[
U_j(u,v) = U_j(u,v) \sum_{m \in M} \sum_{n \in N} \frac{P_j(u - mD_2, v - nD_2)}{f_z} \exp \left\{ \frac{-jk}{2f_z} \left[ (u - mD_2)^2 + (v - nD_2)^2 \right] \right\}.
\] (4)

This field is propagated to the sensor at a distance \( z_3 \), and denoted as \( U_f \). The intensity captured by the sensor is

\[
I_{\text{coherent}}(t,w) = |U_f|^4 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{w}{z_3} + \frac{nD_2}{f_z} \right) \right\} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{t}{z_3} + \frac{mD_2}{f_z} \right) \right\} h'(u,v)^2.
\] (5)

This is the sensor data for a coherent imaging system, when the illumination may be a coherent or a partially coherent source, and the object not fluorescent. The analysis up to this part was briefly discussed in our prior work [4].

2.2 Incoherent Imaging

When we consider an incoherent system, such as with a fluorescent object having intensity \( I_o \), the sensor data can be written as \( \langle U_o(\xi',\eta') \rangle = I_o(\xi',\eta') \delta(\xi' - \tilde{\xi}, \eta' - \tilde{\eta}) \). The intensity at the sensor plane is then given by

\[
I_{\text{incoherent}}(t,w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{w}{z_3} + \frac{nD_2}{f_z} \right) \right\} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{t}{z_3} + \frac{mD_2}{f_z} \right) \right\} h'(u,v)^2.
\] (6)

We introduced this portion of our analysis briefly in [6].

2.3 Coherent and Incoherent Imaging versus System Response

When we consider an incoherent plenoptic system with an impulse as the object, we substitute \( I_o(\xi,\eta) = \delta(\xi,\eta) \) in equation (6). If we consider a coherent plenoptic system, such as one being tested with a laser point source, where the object is an impulse, we substitute \( U_o(\xi,\eta) = \delta(\xi,\eta) \) in equation (5). Both give us the same equation,

\[
I_{\text{coherent}}(t,w) = I_{\text{incoherent}}(t,w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{w}{z_3} + \frac{nD_2}{f_z} \right) \right\} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -jk \left[ \frac{1}{z_3} - \frac{1}{f_z} \right] \left( \frac{t}{z_3} + \frac{mD_2}{f_z} \right) \right\} h'(u,v)^2.
\] (7)
Thus, we see that the sensor intensity response for an impulse is the same for both the coherent and incoherent cases of the plenoptic system. The above equality assures us that light capture in a plenoptic system is not very different from a general system and suggests the possibility of characterizing a plenoptic system response using the sensor response data for coherent impulses such as laser point sources.

We call the intensity impulse response for such a plenoptic system a Pupil Image Function (PIF). This serves to avoid confusion with the general term Point Spread Function (PSF), which is often assumed to be an Airy Disk. The PSF is indeed an Airy disk for a conventional imaging system with a circular pupil. But for a plenoptic system, the response can only be measured at the sensor after the lenslet array and often resembles an image of the pupil of the main imaging lens.

The PIF for an impulse located at \((\xi, \eta)\) is equal to the intensity captured by the sensor as defined in equation (7). The PIF itself is a 4-dimensional function dependent on both the object and sensor coordinates,

\[
PIF_{\xi', \eta'}(t, w, \xi', \eta') = \frac{e^{jk\xi}e^{jk\eta}}{f\lambda z/M^2} \exp \left( \frac{jk}{2z_1} \left( t^2 + w^2 \right) \right) \sum_m \sum_n \exp \left[ -\frac{jk}{2f_2} \left( (mD_2)^2 + (nD_2)^2 \right) \right] \\
\int \int dudv P_2(u-mD_2, v-nD_2) \exp \left[ \frac{jk}{2} \left( \frac{1}{z_2} + \frac{1}{z_3} - \frac{1}{f_2} \right) \left( u^2 + v^2 \right) \right] \\
\exp \left[ -jk \left( \frac{t}{z_2} \frac{mD_2}{f_2} + \frac{w}{z_3} \frac{nD_2}{f_2} \right) \right] h'(u-\xi', v-\eta') \right].
\]

We use the PIF or the Pupil Image Function to characterize plenoptic systems.

3. SYSTEM RESPONSE AND ITS APPLICATIONS

We simulated a system having a main lens and a lenslet array with focal lengths 50mm and 0.87mm, and lens diameters 5mm and 300\(\mu\)m respectively. We assumed an imaging wavelength of 630nm. We used the preceding analysis and explored the impact of some of the parameters in the above equations on the system response, PIF. We assume a single on-axis lenslet and show the response for only one lenslet, however, the simulation is equally applicable for multiple lenslets. We assume a single on-axis impulse at the object plane and simulate the impulse response, IPR, at the plane of the lenslet array. We further propagate the beam to the plane of the sensor and obtain the PIF captured at the sensor.

3.1 System Response for different wavelengths

![Figure 2](image-url)

Figure 2. (a) to (c) show \(|IPR|^{0.2}\) simulated at the plane of the microlens array, with the effect of cropping to the extent of the shape and size of an on-axis lenslet for wavelengths 450nm, 650nm and 850nm respectively. (d) to (f) show \(|PIF|\) system response simulated at the sensor for the same.

Figure 2(a) shows the magnitude of the impulse response (IPR) of the main lens at the plane of the microlens array. The IPR shows that the response of the system until the plane of the lenslet array appears like an Airy disk, which is typical of a conventional system. But in a plenoptic system, this response is further cropped by the extent of the lenslet. This is
shown in Figure 2(a) by a circular cut-off corresponding to the shape and size of the on-axis lenslet. When this field is further propagated beyond the lenslet to the sensor plane, we obtain the corresponding overall system response, or the PIF. The magnitude of the PIF is shown in Figure 2(d). As expected, the simulated PIF resembled the shape of the circular pupil of the main lens. We have shown results with a pentagon shaped pupil in [4]. The rings inside the circular PIF are an effect of diffraction. These figures are displayed on a stretched scale for better visibility of detail and contrast.

In order to explore the effect of the wavelength parameter, \( \lambda \), in the above equations, we simulated the IPR and the PIF for wavelengths 450nm, 650nm and 850nm, shown in Figure 2(a) to (c), and Figure 2(d) to (e) respectively. The FWHM of the Airy Spot in our simulations increased with the wavelength. This is in accordance with expectations, since the spot size at the plane of the lenslet array should be proportional to conventional resolution which equals \( 1.22 \lambda f / D \).

The Airy disk in the IPR was cropped to the same extent by the limits of the circular lenslet for the cases of all wavelengths, but, since the IPR itself is different for different wavelengths, this affected differences in the resultant overall system response, i.e. the PIF at the sensor. The size or magnification of the PIF remained the same for all wavelengths. But the diffraction rings of the PIF became coarser as the wavelength increased resulting in fewer, thicker diffraction rings and a more choppy-looking PIF. Since the coarseness of diffraction rings is greater at longer wavelengths, the details of the PIF can then be captured by fewer and larger pixels. Therefore, if a system is designed to work over a broad range of wavelengths, the pixel size and sampling are best determined by the shortest wavelength.

### 3.2 System Response for different axial distances, \( z_1, z_2 \) and \( z_3 \)

Figure 3. (a) to (h) show \( |\text{IPR}|^{0.2} \) as simulated at the plane of the microlens array, along with cropping by the diameter of a single on-axis lenslet and (i) to (p) show \( |\text{PIF}|^3 \) for varying amounts of object defocus, where \( z_1 = 100\text{mm}, 101\text{mm}, 103\text{mm}, 105\text{mm}, 107\text{mm}, 110\text{mm}, 115\text{mm} \) and 120mm respectively.
Next we kept the wavelength constant at 630nm. The object distance $z_1$ was allowed to vary while keeping all other parameters in the preceding equations constant. Figure 3(a) to (h) shows the magnitude of the IPR at the plane of the microlens array, along with cropping by the diameter of a single on-axis lenslet for varying amounts of object defocus, where $z_1 = 100\text{mm}, 101\text{mm}, 103\text{mm}, 105\text{mm}, 107\text{mm}, 110\text{mm}, 115\text{mm}$ and $120\text{mm}$ respectively. The figures (i) to (p) show the PIF for the same.

The system was designed to be perfectly focused when $z_1 = 100\text{mm}$, as seen in Figure 3(a) and (i). The IPR at the plane of the lenslet array appeared like an Airy Spot and got cropped by the extent of the lenslet. The system response at the sensor plane, the PIF, appeared like an image of the circular pupil, seen in Figure 3(b).

As the amount of object defocus changed, a defocused version of the IPR at the plane of the lenslet array was cropped and propagated further. For smaller amounts of defocus such as $z_1 = 101\text{mm}$ and $103\text{mm}$ in Figure 3(j) and (k), the PIF still appeared like a circular image of the pupil at about the same magnification as for the design distance. But as $z_1$ increased to $105\text{mm}$ and beyond, the diffraction rings became thicker, un-equally spaced and prominent. The magnification of the circle in the PIF also started to reduce.

![Figure 3](http://proceedings.spiedigitallibrary.org/proceedings?paperid=86671L6)

Figure 4. (a) to (h) show $|\text{IPR}|^{0.2}$ as simulated at the plane of the microlens array, along with cropping by the diameter of a single on-axis lenslet and (i) to (p) show $|\text{PIF}|^3$ for varying distance $z_2 = 100\text{mm}, 101\text{mm}, 102.8\text{mm}$, $104.5\text{mm}, 106.1\text{mm}, 108.3\text{mm}, 111.5\text{mm}$ and $114.3\text{mm}$ respectively.

The system was designed to be perfectly focused when $z_2 = 100\text{mm}$, as seen in Figure 4(a) and (i). Varying the distance between the main lens and the lenslet array, $z_2$, while maintaining all other parameters constant allowed us to simulate the PIF and IPR as shown in Figure 4 for $z_2 = 100\text{mm}, 101\text{mm}, 102.8\text{mm}, 104.5\text{mm}, 106.1\text{mm}, 108.3\text{mm}, 111.5\text{mm}$, and $114.3\text{mm}$ which matched the corresponding object defocus distances in Figure 3 via the lens equation for the first imaging sub-system. The PIF results in Figure 3 and Figure 4 were closely similar, although not an exact match. There
are some differences between the configuration of moving an object further from the main lens, versus moving the lenslet array further from the main lens, for instance, the second sub-system is also influenced to some extent by changing $z_2$. But the marked similarity in the PIF results in Figure 3 and Figure 4 indicated that defocus in the first sub-system via error in $z_1$ and $z_2$ both produced similar response consisting of prominent, un-equally spaced and coarse rings in the PIF at the sensor plane. If the change in $z_2$ affected a larger change in the defocus of the second sub-system we would expect to see similarities between the PIFs in Figure 4 and Figure 5, which is discussed next.

![PIF images](image.png)

**Figure 5 (a) to (f) show |PIF| for varying amounts of sensor defocus, where $z_3 = 0.87\text{mm}, 0.9\text{mm}, 0.92\text{mm}, 0.95\text{mm}, 1\text{mm} \text{ and } 1.1\text{mm}$ respectively.**

Figure 5(a) to (f) shows the magnitude of the PIF when we varied the distance of the sensor from the lenslet array as $z_3 = 0.87\text{mm}, 0.9\text{mm}, 0.92\text{mm}, 0.95\text{mm}, 1\text{mm} \text{ and } 1.1\text{mm}$ respectively, affecting only the second imaging sub-system. The system is perfectly focused when $z_3 = 0.87\text{mm}$ as shown in Figure 5(a). For small amounts of defocus where $z_3$ changed from 0.9mm to 0.95mm, seen in Figure 5(b) to (d) the pupil image function became blurred at the edges, there was energy spilling over beyond the circular PIF and the edges of the circle lost their sharpness. As the defocus increased further the diffraction rings were also affected. Note that the size of each sub-image in Figure 5 is not the same. The size of Figure 5(a) to (h) is 361µm, 370µm, 379µm, 391µm, 411µm and 453µm respectively, indicating that the PIF also increased somewhat in magnification as the sensor moved further away from focus. The IPR is not shown in this figure because changing the sensor distance does not affect the IPR at the plane of the lenslet array which is a function of the first imaging sub-system.

### 3.3 Inference from system response dependence on axial distances and wavelengths

The results in the above sections suggested that aberrations such as defocus affect a change in the response of the system. Monitoring the system response or the PIF is a good way to evaluate and characterize the system. In our simulations, prominent diffraction rings marked defocus in the first imaging sub-system, while defocus in the sensor placement affected a spill-over or blur around the edges of the PIF. These simulations also enable us to consider exploring defocus tolerances [4] which maintain a certain acceptable amount of diffraction rings or acceptable demagnification, or spill-over in these axial positions.

As indicated by the simulations with varying wavelengths, it may be necessary to choose the sampling at the sensor plane in accordance with the fineness of diffraction rings in the plenoptic system. Wavelength is one of the factors which affect this. Other parameters include the focal lengths and diameters of the main-lens and lenslet which will be explored in future publications.

### 4. CONCLUSIONS

As the prospect of applying plenoptic systems to more complex and demanding systems like microscopes and high precision cameras increases, it is necessary to develop hardware and software which take into consideration the effects of diffraction in these systems. We propose an analysis and a simulation tool which can be used to predict the effect of varying different parameters of a basic plenoptic system such as axial distances, wavelengths, focal lengths, lens
Our simulation experiments for varying axial positions of the elements of a plenoptic system showed blurring and ringing in the optical response function. Our simulations over different wavelengths show that the shortest wavelengths require finer and denser pixel sampling. Such information can be used for better system design and optimization, characterization, implementation and calibration of plenoptic systems.

5. REFERENCES